Stability of active materials: the role of microstructure

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With the term active materials (or equivalently smart materials) we refer to materials that involve interactions between mechanical, electromagnetic and electronic properties. They are of increasing importance in modern engineering in a wide range of potential applications as sensors, actuators etc. Their microstructure is crucial in understanding their (typically nonlinear) response where stability phenomena play a determinant role. In this talk we present two different applications: Magneto-rheological Elastomers (MRE's) and Nematic Elastomers (where Liquid Crystals constitute a special case).

The first application pertains to MREs are ferromagnetic particle impregnated rubbers whose mechanical properties are altered by the application of external magnetic fields. We present a combined experimental and theoretical study of the macroscopic response of a particular MRE consisting of a rubber matrix phase with spherical carbonyl iron particles cured in the presence of strong magnetic fields leading to the formation of particle chain structures and thus to an overall transversely isotropic composite. Two different continuum mechanics approaches are used: a direct approach using an Eulerian description (current configuration) and a fully coupled Lagrangian (reference configuration) variational principle, showing that both give the same governing equations and interface conditions. We then propose a transversely isotropic energy density function that is able to reproduce the experimentally measured magnetization, magnetostriction and simple shear curves under different pre-stress, initial particle chain orientations and magnetic fields. Microscopic mechanisms are also proposed and simulated using numerical (FEM) calculations to explain i) the counterintuitive effect of dilation under zero or compressive applied mechanical loads for the magnetostriction experiments and ii) the importance of a finite strain constitutive formulation even at small magnetostrictive strains. We subsequently propose a device made of a thin MRE film on a non-magnetic isotropic pure polymer/gel substrate, which is destabilized by a transverse magnetic field. An analytical solution to this problem is presented in 2D, which shows that for a large range of substrate stiffness there is a finite-wavelength buckling mode induced by the magnetic field. Moreover, the critical magnetic field can be substantially reduced in the presence of a compressive prestress of the assembly, thus opening the possibility of controlling haptic interfaces with low magnetic fields.

The second application pertains to Nematic Elastomers, which are rubber-like solids formed by cross-linking polymeric chains that include liquid crystal molecules. The electromagnetic/mechanical interactions between the elasticity of the network and the alignment of liquid crystals lead to peculiar properties that make nematic elastomers a promising material for applications like fast soft actuators and artificial muscles. A general theory for nematic continua that accounts for electro-mechanical coupling is presented, based on a Lagrangian variational formulation with a potential energy that depends on four independent vector fields: the local mechanical displacement, the liquid crystal director, the specific polarization and the electric displacement potential. Variations of the potential energy with respect to each one of these variables lead to the governing mechanical and electric equations. An application of this general theory is presented next to the study of the stability of a Liquid Crystal (special case of these materials, for the case of negligible shear and bulk elastic moduli) confined between two parallel plates and subjected to an electric field perpendicular to these plates. As the electric field reaches a critical value, the nematic directors, which are initially parallel to the bounding plates, change orientation. This transition, noted in the literature as Freedericksz transition, is treated here as a bifurcation problem. We present a fully three-dimensional analysis for the Freederickz transition of the Twisted Nematic Device (TND) that is commonly used in Liquid Crystal Display (LCD) monitors. Specifically, we study a liquid crystal layer – that is infinite in 2 directions but finite in the third – whose directors, anchored at the boundaries, are parallel to the two confining plates and arranged in a helix (twist). The liquid crystal is subjected to a uniform external electric field in the direction normal to the plates. As the electric field reaches a critical value, the nematic directors, which are initially parallel to the bounding plates, change orientation. Starting from the Frank-Oseen energy for liquid crystals and adding the electrical energy contribution, we find the energy functional for the three-dimensional TND. Given the non-uniform principal solution (directors depend on their position relative to the plate boundaries) we solve the problem numerically using the finite element method, in conjunction with the Fourier transform along the directions parallel to the plates. The nature of the critical mode (i.e. mode corresponding to the lowest value of the electric field), whether it is local (corresponding to finite wavenumbers) or global (corresponding to vanishing wavenumbers, indicating a solution which is invariant with respect to the direction parallel to the plate and depending only on the thickness coordinate), depends on the parameters of the Frank-Oseen energy. For the special case of no twist, an analytical solution of the problem is possible that not only allows for the verification of the FEM algorithm, but also provides valuable insight on the problem at hand.